Entrepreneurship, On-the-job Search and Informal Jobs

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Abstract

This paper develops a labour market matching model à la Pissarides (2000), with on-the-job search, extended to both the informal sector and heterogeneous entrepreneurial ability. In this model, the entrepreneurial ability affects job productivity, all unemployed start their job search in the official sector, and workers employed in the informal sector try to move into the official one. Firms become heterogeneous in productivity, thus providing a new solution to the problem of determining a mixed allocation of vacant jobs between the regular and the irregular sector. Because of the on-the-“informal”-job search assumption, the effect of labour market tightness on unemployment rate is a priori ambiguous, thus showing the intricate relationship found in the literature between unemployment and underground employment.

JEL Classification: E26, J23, J24, J63, J64, L26

Keywords: entrepreneurship, on-the-job search, unemployment, matching models, shadow economy, hidden economy, underground economy, informal jobs

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1. Introduction

Usually in matching-type models, the supply side of the labour market is well-characterized, whereas the demand side is described only by the free-entry (or zero-profit) condition used to determine the vacancies posted by firms. For instance, two recent and influential papers (Albrecht et al., 2009; Boeri and Garibaldi, 2006) model the heterogeneous ability (productivity) of workers. In these matching models, the ability of workers affects job productivity. However, analogous reasoning should also be applied to the (entrepreneurial) ability of entrepreneurs. Nevertheless, the additional extension of matching models to entrepreneurship is somewhat innovative. Indeed, only Fonseca et al. (2001) and Pissarides (2002) have pursued this kind of analysis, but they have ignored the fact that entrepreneurial ability affects job productivity. In particular, in their models the entrepreneurship only affects the entry costs. This is a shortcoming for the benchmark macroeconomic model of the labour market since entrepreneurship is crucial for spurring structural change, new firms, innovations and employment growth (Belke et al., 2003). Entrepreneurship is in fact like a production factor, the so-called “entrepreneurial capital” (Audretsch and Keilbach, 2004; Audretsch, 2007), and then it realistically affects job productivity. Furthermore, in many analyses the on-the-job search assumption is often neglected. Although the equilibrium unemployment theory obtained in matching models with on-the-job search is not significantly different from that obtained without it (Pissarides, 2000), the on-the-job search assumption is quite realistic since the match is not the end of an individual’s search. Indeed, the main aim of search activity is often to find “the best partner not a simple partner”. This is all the more relevant in the underground economy, since the underground sector is seen as a backward and less productive component of the economy in which the wages are lower. However, because of high unemployment and skill-mismatch problems, it is often not convenient to reject informal job offers and wait for a job offer from the official sector. But, once hired, workers try to move from the informal sector to the official one.

This theoretical paper belongs to a growing literature which has just put together the underground economy theory with the benchmark macroeconomic model of the labour market, i.e. the matching model (see e.g. Bouev, 2002, 2005; Boeri and Garibaldi, 2002, 2006; Kolm and Larsen, 2003; Fugazza and Jacques, 2004; Albrecht et. al., 2009; Bosch and Esteban-Pretel, 2009). Among this literature, this model is perhaps most related to that of Boeri and Garibaldi (2002) and Bosch and Esteban-Pretel (2009) who make use of a matching model extended to the underground sector with the possibility
of direct transitions from informal into formal employment. However, neither of them take into account the heterogeneous entrepreneurial ability on the demand side. Furthermore, in this model, matching first takes place for official jobs and then for informal jobs, which means each worker prefers to work in the official sector (for instance, in order to be entitled to a pension).

More precisely, this paper adopts a matching model à la Pissarides (2000) with on-the-job search, extended to the underground sector and to heterogeneous entrepreneurial ability. These three features/extensions together have interesting analytical consequences. The first is that firms become heterogeneous in productivity, thus providing a new solution to the problem of finding an interior equilibrium, where vacant jobs are allocated to both the regular and the hidden sector. The second consequence is that the effect of labour market tightness on unemployment rate is a priori ambiguous, thus showing the intricate relationship found in the literature between unemployment and underground employment (Tanzi, 1999; Giles and Tedds, 2002). Indeed, the sign of this relationship depends on various assumptions regarding the position of shadow employment in the Labour Force Statistics (Boeri and Garibaldi, 2002, 2006) since very heterogeneous workers compose the labour force of hidden economy (Tanzi, 1999).¹ In this model, the probability to find a job in the official sector is in fact a further job destruction rate for the informal jobs. Hence, an increase in labour market tightness may not reduce the “overall” unemployment rate.

The rest of this paper is organized as follows: sections 2, 3 and 4 present a two-sector matching model with on-the-job search and heterogeneous entrepreneurs; section 5 provides the usual exercises of comparative statics; while section 6 provides a quantitative evaluation of the model; section 7 concludes.

2. The model: building blocks

We propose a matching model with on-the-job search and informal jobs. The economy is populated by a [0,1] continuum of (heterogeneous) entrepreneurs (one-job firms) and a [0,1] continuum of workers who prefer to work in the official sector in order to be

¹ The intricate relationship between unemployment and the hidden economy is a subject of great interest and currently open to debate even in the matching literature. Indeed, according to Bouvé (2002, 2005), scaling down the unofficial sector can lead to a decrease in the level of unemployment; whereas according to Boeri and Garibaldi (2002, 2006), attempts to reduce, in the first place, shadow employment will result in higher open unemployment (although they consider the two phenomena as “two sides of the same coin”). Bosch and Esteban-Pretel (2009) focus on the role of job destruction rate. According to their matching model, policies that reduce the cost of formality (or those that increase the cost of informality) produce an increase in the share of formal employment while also reducing unemployment because the reallocation between formal and informal jobs has non-neutral effects on the unemployment rate, since informal jobs report much higher separation rates.
entitled to a pension. Therefore, we assume that the job-seekers in this economy are both the unemployed workers and the workers employed in the informal sector. Hence, workers employed in informal jobs try to move into the official sector. The matching frictions on the official side of the labour market are captured by a popular constant returns to scale matching function (Petrongolo and Pissarides, 2001). It follows that the labour market tightness is given by:

\[ m_r = m\{v_r, u + n_s\} \Rightarrow \theta = \frac{v_r}{u + n_s} \]

where \( v_r \) is the total number of vacancies supplied by official firms, \( u \) is the unemployment rate and \( n_s \) is the shadow employment, i.e. the measure of employed job-seekers. The number of vacancies supplied by the formal and the informal sector are \( v_i \), with \( i \in \{r, s\} \), where \( r = \text{regular} \) and \( s = \text{shadow} \) (whereas, \( n_i \) are the employment rates since the labour force is normalized to unity). Workers who fail to find a job in the official sector search in the informal one, which means that matching initially takes place for official jobs and then for informal jobs. Indeed, this economy can be seen as comprised of two separate theatres of search activity: once workers fail to find an official job, they immediately enter the underground sector and search for employment. Hence, the number of individuals prepared to accept informal jobs is equal to \( 1 - m_r \).

### SEARCH PATH

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job-seeker ---- search in the official sector ---- search in the informal sector ---- unemployed

employed --- unemployed
employed
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With respect to informal jobs, we assume frictionless matching (i.e. there is a spot-market for informal jobs), because of the absence of skill-mismatch problems. The number of informal matches is thus given by:

\[ m_s = \min\{1 - m_r, v_i\} \]

where \( 1 - m_r \) and \( v_i \) refer to the number of workers seeking informal jobs and the number of informal vacancies, respectively. Following Dulleck et al. (2006), we can

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2 Time is continuous, and individuals are risk neutral and infinitely lived. We neglect possibilities of moonlighting, so workers can perform only one activity at a time. We also neglect the endogenous decision of individuals to either become entrepreneurs or workers because it is widely discussed in the matching literature (see Fonseca, Lopez-Garcia and Pissarides, 2001; Pissarides, 2002, and Uren, 2007). This framework can thus be seen as a following step.

3 Note that the number of official matches \( m_r \) is equal to the number of job-seekers \( (u + n_s) \) multiplied by the probability of finding an official job.
think of informal/unskilled jobs as “hamburger-flipping positions which can be found at virtually no cost at all”. In order to rule out the trivial (unrealistic) case in which $u = 0$, we assume that $1 - m_r > v_s$, so that $\min\{1 - m_r, v_s\} = v_s$. The rationale is the following: since the informal firms know the preferences of workers, i.e. the “timing” of the search (search path), there will be an under-supply of informal vacancies. Hence, the probability of filling a vacancy in the informal sector is $\min\{1 - m_r, v_s\} / v_s = 1$, and thus the bargaining power of workers who search for an informal job is zero. As a result, the informal wage is equal to the minimum wage, i.e. the unemployment benefit, $w_s = b$

As stated above, the job search takes place in two steps: at first, all unemployed workers search in the official sector, and afterwards (in the case of failure) they search in the informal one:

$$rU = b - k + g(\theta) \cdot [W_r - U] + \left[1 - g(\theta)\right] \cdot \frac{v_s}{1 - m_r} \cdot [W_s - U]$$

where $U$ is the value of searching for a job; $k$ is the search cost which reflects the search effort (implicitly assumed) of unemployed workers (see later); $g(\theta)$ is the probability of finding an official job; and $[v_s / (1 - m_r)] < 1$ is the probability of finding an informal job (conditional to failure in the search of a formal job). Obviously, $g'(\theta) > 0$, $g''(\theta) < 0$, and $\lim_{\theta \to 0} g(\theta) = 0 (\infty)$.

The workers employed in the informal jobs try to move into the official sector:

$$rW_s = w_r + \delta \cdot [U - W_r]$$

$$rW_s = b + (\delta + \rho) \cdot [U - W_s] + g(\theta) \cdot [W_r - W_s]$$

where $W_i$ is the value for being employed; $w_r$ is the official wage; $\rho$ is the exogenous instantaneous probability of a firm being discovered (and destroyed) as unregistered; and $\delta$ is the exogenous destruction rate. It is implicitly assumed that informally employed and unemployed workers search with the same intensity and that they are equally good at finding jobs. Therefore, official jobs arrive to each job-seeker at the same rate, which is equal to $g(\theta)$.

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4 Wages in the informal sector do not depend on outside market conditions. This property is a key feature of matching models with on-the-job search, and it was already noted by Pissarides (2000) and Boeri and Garibaldi (2002).

5 The “timing” of the search (namely, unemployed workers never search in the informal sector before failing to find a job in the official one) implies that the events are independent.
3. Entrepreneurship

Entrepreneurs are born with a specific entrepreneurial ability \((x)\) which is drawn from a known distribution \(F: [x_{\text{min}}, x_{\text{max}}] \rightarrow [0,1]\). Entrepreneurs can either operate regularly or against the tax regulations. The value of a vacancy and a filled job in each sector is given by the following Bellman equations:

\[
\begin{align*}
 rV_r &= -c_r + q(\theta) \cdot [J_r - V_r] \\
rJ_r &= xp - w_r - \tau + \delta \cdot [V_r - J_r] \\
rV_s &= -c_s + J_s, \text{ with } J_s = \frac{xp\phi - b}{r + \delta + \rho + g(\theta)}
\end{align*}
\]

where \(c_i\) is the entry cost; \(x\) is the entrepreneurial ability; \(p\) is the productivity of the match; while \(q(\theta)\) refers to the probability of filling a vacancy in the official sector, with \(q'(\theta) < 0, \ q''(\theta) > 0\), and \(\lim_{\theta \to 0^+} q(\theta) = \infty(0)\). As in Bosch and Esteban-Pretel (2009), evading production tax \((\tau)\) implies that irregular firms can only take advantage of a fraction \(\phi \in (0,1)\) of the productivity of the match.\(^6\) Note that for the informal firms, the probability of finding a job in the official sector is a further job destruction rate. As usual, official wage solves this problem:

\[
W_r = \arg \max \left[ W_r - U \right] \cdot (J_r - V_r)^{1-\beta} \Rightarrow \left( W_r - U \right) = \frac{\beta}{(1-\beta)} \cdot (J_r - V_r)
\]

where \(\beta \in (0,1)\) is the surplus share for labour. In equilibrium, the optimal search effort of unemployed workers requires that the search cost must be equal to the expected net gain resulting from the search:

\[
k = g(\theta) \cdot [W_r - U] + [1 - g(\theta)] \cdot \frac{V_r}{1 - m_r} \cdot [W_s - U]
\]

hence, the search cost depends on the number of vacancies, both official and informal, since the higher the number of vacancies, the larger the search effort. The previous condition implies that \(rU = b\). Hence, the wage and the surplus in the official sector are given by:

\[
\begin{align*}
(W - \frac{b}{r}) = \frac{\beta}{1-\beta} \cdot (J - V) \Rightarrow W_r = (1 - \beta) \cdot b + \beta \cdot (xp - \tau - rV) \\
rS_r = rJ_r - rV_r + rW_r - b \Rightarrow S_r = \frac{xp - \tau + c_r - b}{r + \delta + q(\theta) \cdot (1-\beta)}
\end{align*}
\]

\(^6\) Hence, our notion of underground employment is one of low productivity jobs. Indeed, there is a negative correlation between informal-sector employment and education level within countries (cf. Albrecht et. al., 2009).
Note that in this model the entrepreneurial ability affects the job productivity. This key novelty make heterogeneous the wages within the official sector.

The cut-off arbitrage condition which defines a threshold level of entrepreneurial ability, \( R \in [x_{\min}, x_{\max}] \), such that the marginal entrepreneur is indifferent to operating in the informal or formal sector is the following:

\[
V_r (\theta, x = R) = V_r (\theta, x = R)
\]  

(5)

From the Bellman equations, on the demand side it is straightforward to obtain:

\[
rV_r (x, \theta) = \frac{q(\theta) \cdot (1 - \beta) \cdot (xp - \tau - b) - c_r \cdot (r + \delta)}{r + \delta + q(\theta) \cdot (1 - \beta)}
\]

(6)

\[
rV_r (x, \theta) = -c_r + \frac{xp\phi - b}{r + \delta + \rho + g(\theta)}
\]

(7)

Hence, \( R \) can thus be derived:

\[
R = \frac{\Omega(\theta) \cdot (\tau + b) + c_r \cdot (r + \delta) - c_s - b}{\Lambda(\theta) - \frac{p\phi}{\Gamma(\theta)}}
\]

(8)

with \( \Omega(\theta) \equiv q(\theta) \cdot (1 - \beta) / \Gamma(\theta) \), \( \Gamma(\theta) \equiv r + \delta + q(\theta) \cdot (1 - \beta) \), and \( \Lambda(\theta) \equiv r + \delta + \rho + g(\theta) \). The threshold value \( R \) is a special \( x \), so that it must be positive since \( x \geq x_{\min} \geq 0 \). Hence, the r.h.s of equation (8) must also be positive. Sufficient conditions for the positivity of \( R \) (see Appendix A for the mathematical details) imply that the intercept of (6) is more negative than the intercept of (7), and that the slope of \( rV_r (x, \theta) \) is steeper than the slope of \( rV_s (x, \theta) \).

As a result,

**Remark 1.** Official jobs are manage by the more able entrepreneurs. This result is consistent with the assumption that informal jobs are low productivity jobs.

Furthermore, from the restrictions which ensure the positivity of \( R \), it follows that \( \partial R / \partial \theta > 0 \) (see Appendix A). Indeed,

\[
\text{lim}_{\theta_{\rightarrow 0}} R = \frac{\tau + b - c_s - b}{r + \delta + \rho} > 0, \text{ and } \text{lim}_{\theta_{\rightarrow +\infty}} R = \frac{c_r - c_s}{\rho} \rightarrow +\infty \text{ if realistically } c_r > c_s. \]

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\[7\] Recall that \([J_r - V_r] = (1 - \beta) \cdot S_r.\]

\[8\] Indeed, the entry cost in the hidden sector is very low, since ease of entry is often one of the criteria used to define the informal sector (Gërxhani, 2004). By contrast, the start-up cost in the official sector is often very high because of excessive regulations, administrative burdens, licence fees, bribery (Bouev, 2005).
Equation (8) has been built for \( R \in [x_{min}, x_{max}] \) so that its intercept must be lower than \( x_{max} \). The key property that \( \partial R / \partial \theta > 0 \) is very intuitive since the higher the labour market tightness, the more difficult it is to fill a regular vacancy and thus more entrepreneurs enter the informal sector.

The entrepreneurs’ indifference condition (5) implies that the share of entrepreneurs (either posting a vacancy or producing) in the hidden sector is \( F(R) = n_r + v_r \), while the share \( 1 - F(R) = n_r + v_r \) runs a business in the official sector. Using the summing-up condition on the supply side, i.e. \( u = 1 - n_r - n_r \), the aggregate definition of labour market tightness thus becomes the following:

\[
\theta = \frac{\{[1 - F(R)] - n_r\}}{u + n_r} = \frac{\{[1 - F(R)] - n_r\}}{1 - n_r}
\]

from which it is possible to obtain that \( \partial \theta / \partial R < 0 \) (see Appendix B for the mathematical details), with \( \lim_{R \to x_{min}} \theta = 0 \), since \( F(R) \to 1 \) but the vacancies cannot be negative, and \( \lim_{R \to x_{max}} \theta = 1 \), since \( F(R) \to 0 \). Hence, equations (8) and (9) can be represented in the same diagram with axes \([\theta, R]\), as in fig. 2.

As a result,

**Remark 2.** A unique couple of \((\theta, R)\) exists in the model. 9

Therefore, the two key variables of the model, i.e. the equilibrium value of labour market tightness and the threshold value of entrepreneurial ability can thus be obtained.

### 4. (Un)Employment

Steady-state employment and unemployment rates are obtained by considering the flows into and out of employment, and the summing-up condition or unemployment identity. The equations determining the employment rates \( n_r, n_i \) are given by:

\[
g(\theta) \cdot (1 - n_r) = \delta \cdot n_r \tag{10}
\]

\[
v_r = \left[ \delta + \rho + g(\theta) \right] \cdot n_r \tag{11}
\]

since \( 1 - n_r = u + n_r \) and \( \min\{1 - m_r, v_r\} / (1 - m_r) \cdot (1 - m_r) = v_r \). 10

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9 This is anything but a trivial result. Indeed, as claimed by Bouev (2005), the most important weakness in the underground economy theory is represented by the restrictive assumptions required to find an interior equilibrium in which the underground sector coexists with the regular one in the long run.

10 Recall that the informal vacancies are immediately filled when the job-seekers fail to find a job in the official sector. Hence, in the beginning there is difference between informal vacancies and informal jobs.
Solving for the employment rates in steady state, we obtain:

\[ n_s = \frac{g(\theta)}{\delta + g(\theta)} \]  

\[ n_s = \frac{v_s}{[\delta + \rho + g(\theta)]} \]  

The total share of entrepreneurs in the informal sector is \( F(R) = n_s + v_s \). Hence, we get:

\[ n_s = \frac{F(R)}{[\delta + \rho + g(\theta)] + 1} \]  

\[ v_s = \frac{F(R) \cdot [\delta + \rho + g(\theta)]}{[\delta + \rho + g(\theta)] + 1} \]  

Finally, using the summing-up condition or unemployment identity, we obtain the unemployment rate of this economy:

\[ u = 1 - \frac{g(\theta)}{\delta + g(\theta)} - \frac{F(R)}{[\delta + \rho + g(\theta)] + 1} \]  

where \( u = u_s + u_i \) is the “overall” unemployment rate.

5. Comparative Statics: changes in productivity

In this section we present the usual comparative statics exercises which summarize the qualitative predictions of the model and provide insights to the behaviour of the model (in the steady state) following changes in the productivity of the match.

The equilibrium of the model is composed of two key variables, \( \theta \) and \( R \), which satisfy the following two equations:

\[ R = \frac{\Omega(\theta) \cdot (r+b) + c_s \cdot \left( \frac{r+\delta}{\Gamma(\theta)} \right) - c_s - \frac{b}{\Lambda(\theta)}}{\Omega(\theta) \cdot p - \frac{p\phi}{\Lambda(\theta)}} \]  

\[ \theta \cdot \delta + g(\theta) \cdot F(R) = [1 - F(R)] \cdot \delta \]  

The remaining important variables of the model, \( (n_s, n_s, v_s, u) \), formal and informal employment, informal vacancies and unemployment rate are given by the following four equations:

\[ n_s = \frac{g(\theta)}{\delta + g(\theta)} \]  

\[ n_s = \frac{F(R)}{\delta + \rho + g(\theta) + 1} \]
Figure 3 illustrates graphically the implication of an increase in productivity, $p$.

**Proposition 1.** An increase in productivity, $p$, reduces the threshold value of entrepreneurial ability and increases the labour market tightness: hence, more entrepreneurs enter the official sector.

The productivity change only affects equation (I). Differentiating it with respect to $p$ yields:

$$
\frac{\partial R}{\partial p} < 0 \quad \text{if} \quad \left[ \Omega(\theta) - \frac{\phi}{\Lambda(\theta)} \right] > 0
$$

since $\Omega'(\theta) < 0$ and $\Lambda'(\theta) > 0$, the negative effect of productivity change on the threshold value of entrepreneurial ability is ensured by the small value of $\phi$. Indeed,

$$
\lim_{p \to 0} \left[ \Omega(\theta) - \frac{\phi}{\Lambda(\theta)} \right] = 1 - \frac{\phi}{r + \delta + \rho} \Rightarrow \phi < r + \delta + \rho.
$$

This means that, in times of economic growth, regular firms benefit more from the increase in productivity, because informal jobs are in fact less productive than regular ones since $\phi < 1$.

**Proposition 2.** An increase in productivity, $p$, increases the probability of finding an official job: hence, it increases the share of official employment but reduces the share of informal employment.

These effects can be obtained in a straightforward manner from equations (III) and (IV). Therefore, the share of formal employment should follow a pro-cyclical pattern (the opposite is true for the share of informal employment).

**Proposition 3.** In turn, the previous effect increases the level of informal vacancies.

This follows from equation (V), since $\partial v_2 / \partial g(\theta) > 0$. Indeed, for the informal firms the probability of finding a job in the official sector is a further destruction rate. Hence, the higher $g(\theta)$, the larger the direct transition of workers from the informal sector to the official one, i.e. the higher the destruction of informal employment $n_s$.

Hence, informal jobs become vacant once again.
**Proposition 4.** In virtue of direct transition of workers from the informal sector into the official one, the net effect of labour market tightness on unemployment is a priori ambiguous.

An increase of the probability of finding a job in the official sector reduces informal jobs but at the same time increases the informal vacancies (see Proposition 2 and 3 together). This latter effect, in turn, further spurs informal employment, as clearly emerges from the steady-state equilibrium condition in the informal sector, i.e. equation (11). As a result, the final effect of jobs composition on the unemployment rate is a priori ambiguous. Indeed,

$$\frac{\partial u}{\partial \theta} = -g'(\theta) \cdot \delta \cdot \frac{F'(R(\theta)) \cdot \left[\delta + \rho + g(\theta) + 1\right]}{\left[g(\theta) \cdot \delta\right]^2} + \frac{F(R) \cdot g'(\theta)}{\left[\delta + \rho + g(\theta) + 1\right]^2}$$

Finally, note that the quantitative results will depend on the distribution of the entrepreneurial ability $F(R)$.

**6. Simulation**

In order to provide a quantitative evaluation of this model, a numerical simulation is performed. In particular, this simulation tries to resolve the Shimer puzzle (2005), i.e. the lack of enough amplification in unemployment and vacancies volatility. Indeed, the standard matching model (i.e. the Mortensen & Pissarides model) fails to reproduce the large volatility in unemployment and vacancies during the business cycle. These variables are in fact much more volatile in the U.S. data than in the calibrated model subject to productivity shocks of a realistic magnitude (Shimer, 2005).11

The comparative statics exercises of the previous section aid in understanding the implications of this simulation.

The baseline specification of the model’s parameters has been drawn from Boeri and Garibaldi (2006), and is described in Table 1 (see end). As in Uren (2007), we compare different steady states as the value of the productivity ($p$) and the job destruction rate ($\delta$) vary. Consistent with the data presented in Shimer (2005), the productivity ($p$) and the job destruction rate ($\delta$) are stochastic and are drawn from a bivariate normal distribution constructed to match the stylised business cycle facts of the U.S. economy. Hence, the mean value of $p$ is normalised to 1 and the standard deviation is set to 0.019.

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11 See Hornstein, Krusell, and Violante (2005), Yashiv (2006) and Mortensen and Nagypál (2007), for exhaustive surveys of this important, huge and recent literature.
whereas the mean of $\delta$ is set to 0.1 and the standard deviation is set to 0.075. Finally, the correlation between these variables is -0.6, by construction.\footnote{The statistics for job vacancies and unemployment are those reported in Shimer (2005) for an Hodrick-Prescott (HP) filter with a smoothing parameter of $10^5$ instead of 1600 as followed by Andolfatto (1996) and Merz (1995). However, as claimed by Hornstein et al. (2005), the choice of smoothing parameter has no impact on the unemployment and vacancy statistics.}

The sectoral choice of entrepreneurs is also solved analytically by the variation of the threshold value $R$. We assume that the idiosyncratic productivity, $F(x)$ distribution is negative exponential in the range $[x_{\text{min}}, x_{\text{max}}]$. Therefore, it takes the form:

$$F(x) = 1 - e^{-x/\lambda}$$

we assume as in Boeri and Garibaldi (2006) that $\lambda = 1$.\footnote{Boeri and Garibaldi (2006) use a negative exponential distribution for the distribution of labour productivity.}

The simulation works as follows: given $p$ and $\delta$, the corresponding values of $R$, $\theta$, $\nu$, and $u$ are calculated. More precisely, productivity and job destruction shocks upset the equilibrium of the economy. The equilibrium (starting) values of entrepreneurial ability threshold and labour market tightness are calculated considering the mean values of $p$ and $\delta$ (i.e. 1 and 0.1, respectively). Hence, $\theta^*$ and $R^*$ are 0.11 and 0.192, respectively (which imply, with an exponential distribution and a lambda parameter equal to 1, a realistic share of informal economy equal to 18%).

This process is repeated 1,000 times and the correlations and the standard deviations of the key variables are calculated. The simulation results are shown in table 2.

The correlations enrich the qualitative predictions of the model. An increase in productivity reduces the threshold value of entrepreneurial ability and increases the labour market tightness, but it discourages the opening of informal vacancies, since they are less productive than the official ones. However, as claimed by the theoretical model, there is a positive correlation between informal vacancies and the probability of finding a job in the official sector. Furthermore, the net effect of productivity on unemployment is negative, thus confirming its counter-cyclical nature.

The model generates significantly greater amplification of shocks than implied by the standard Mortensen-Pissarides model as calibrated by Shimer (2005). In particular, the standard deviation of unemployment in this model is 0.187 which compares to a value of 0.031 in Shimer‘s calibration with both productivity and job separation shocks. More importantly, increasing the volatility of unemployment by a factor of six, the model generates equal volatility than observed in the data, with Shimer (2005) reporting the cyclical volatility of unemployment equal to 0.190. Therefore, as claimed by Busato and
Chiarini (2004) – in a *Real Business Cycle* framework – and by Bosch and Esteban-Pretel (2009) – in a *matching* framework – the underground economy fluctuations help to understand the business cycles, i.e. introducing an underground sector improves the fit of the model to the data.

Finally, the same shocks to productivity and the job separation rate result in a negative correlation between unemployment and labour market tightness (the probability of finding an official job) of $-0.381$ ($-0.544$), corresponding to a negatively sloped Beveridge Curve.

7. Conclusions

This paper develops a labour market matching model à la Pissarides (2000), with on-the-job search, extended to the informal sector and to heterogeneous entrepreneurial ability. In this model, the entrepreneurial ability affects job productivity, all unemployed begin by searching in the official sector, and workers employed in the informal sector try to move into the official one. Three key results emerge from this analysis:

- Firms become heterogeneous in productivity, thus providing a new solution to the problem of determining a mixed allocation of vacant jobs between the regular and the irregular sector;
- Because of the on-the-“informal”-job search assumption, the effect of labour market tightness on unemployment rate is a priori ambiguous, thus showing the intricate relationship found in the literature between unemployment and underground employment;
- These key modifications improve the quantitative properties of the standard matching model, thus providing a possible explanation for the unemployment volatility puzzle.
Appendixes

Appendix A: Properties of equation (8)

Sufficient conditions for the positivity of $R$ are:
\[
\Omega(\theta) \cdot (\tau+b) + c_r \cdot \frac{(r+\delta)}{\Gamma(\theta)} > c_s + \frac{b}{\Lambda(\theta)}
\]  
(A.1)
\[
\Omega(\theta) \cdot p > \frac{p\phi}{\Lambda(\theta)}
\]  
(A.2)

In order to determine the restrictions on the parameters, we calculate the limit of conditions (A.1) and (A.2) for the labour market tightness which goes to zero:
\[
\tau + b - \frac{b}{r + \delta + \rho} > c_s
\]  
(A.1’)
\[
r + \delta + \rho > \phi
\]  
(A.2’)

The proof that $\partial R / \partial \theta > 0$ follows from the previous restrictions. Indeed, since:
\[
\frac{\partial \Omega}{\partial \theta} = \frac{(r+\delta) \cdot q'(\theta) \cdot (1-\beta)}{\Gamma^2} < 0, \quad \frac{\partial \Gamma}{\partial \theta} = (1-\beta) \cdot q'(\theta) < 0, \quad \text{and} \quad \frac{\partial \Lambda}{\partial \theta} = g'(\theta) > 0; \lim_{\theta \to 0} \Omega = 1 \text{ by the l’Hôpital rule, } \lim_{\theta \to 0} \Gamma = \infty, \text{ and } \lim_{\theta \to 0} \Lambda = r + \delta + \rho; \lim_{\theta \to 0} \Omega = 0, \lim_{\theta \to 0} \Gamma = r + \delta, \text{ and } \lim_{\theta \to 0} \Lambda = \infty; \text{ we get:}
\]
\[
\lim_{\theta \to 0} R = \frac{\tau + b - c_s - \frac{b}{r + \delta + \rho}}{p - \frac{p \cdot \phi}{r + \delta + \rho}} > 0, \text{ and, } \lim_{\theta \to 0} R = \frac{c_r - c_s}{0} \to +\infty, \quad \text{if } c_r > c_s
\]  
(A.3)

The numerator of (8) is rising in $\theta$, $\frac{\partial \text{num}[R]}{\partial \theta} > 0$:
\[
\frac{\partial \text{num}[R]}{\partial \theta} = \frac{(r+\delta) \cdot q'(\theta) \cdot (1-\beta)}{[r+\delta+q(\theta)-(1-\beta)]^2} (\tau+b) - c_r \cdot \frac{(r+\delta) \cdot q'(\theta) \cdot (1-\beta)}{[r+\delta+q(\theta)-(1-\beta)]^2} + \frac{b \cdot g'(\theta)}{[r+\delta+\rho+g(\theta)]^2}
\]  
if $c_r > (\tau+b)$  
(A.4)

whereas condition (A.2’) ensures that the denominator of (8) is not increasing in $\theta$ (i.e. the positive term of the denominator of (8) decreases more than the negative one).

Appendix B: Property of equation (9)

The evolution of official employment in terms of the firm’s transition rates is:
\[
\dot{n}_s = [1 - F(R) - n_r] \cdot q(\theta) - \delta \cdot n_r
\]

it follows that:
\[
n_r = \frac{[1 - F(R)] \cdot q(\theta)}{q(\theta) + \delta}
\]

Hence, the labour market tightness – i.e. equation (9) – can be rewritten as:

---

14 The labour market tightness cannot be equal to $\infty$, since the total number of entrepreneurs in the economy is equal to 1 and because of the restriction $u > 0$. Furthermore, the assumption that $\theta = \infty$ is often ruled out (see e.g. Fonseca et al., 2001; Pissarides, 2002) because in that case the value of a vacancy is negative.
\[
\begin{align*}
\left\{1 - \frac{[1 - F(R)] \cdot q(\theta)}{q(\theta) + \delta}\right\} \cdot \theta &= [1 - F(R)] - \frac{[1 - F(R)] \cdot q(\theta)}{q(\theta) + \delta} \\
\left\{\frac{\delta + q(\theta) \cdot F(R)}{q(\theta) + \delta}\right\} \cdot \theta &= \frac{[1 - F(R)] \cdot \delta}{q(\theta) + \delta} \\
\Rightarrow \theta \cdot \delta + g(\theta) \cdot F(R) &= [1 - F(R)] \cdot \delta \\
\end{align*}
\]

(B.1) since \(\theta \cdot q(\theta) = g(\theta)\). Totally differentiating the previous expression yields:

\[
\begin{align*}
d\theta \cdot \delta + d\theta \cdot g'(\theta) \cdot F(R) + g(\theta) \cdot F'(R) \cdot dR &= -F'(R) \cdot dR \cdot \delta \\
\frac{d\theta}{dR} &= -\frac{F'(R) \cdot [\delta + g(\theta)]}{\delta + g'(\theta) \cdot F(R)} < 0 \\
\end{align*}
\]

(B.2) since \(F'(R) > 0\).
References


Figures

Figure 1. Entrepreneurs’ indifference condition

$x = R$

Figure 2. Equilibrium interior solution

$\frac{\partial R}{\partial \theta} > 0$

$\frac{\partial \theta}{\partial R} < 0$

* For simplicity, we assume $x_{min} = 0.$
Figure 3. Increase in productivity
### Tables

<table>
<thead>
<tr>
<th>Matching function</th>
<th>$m_i = v_i^\eta \cdot u_i^{1-\eta}$</th>
<th>Petrongolo &amp; Pissarides (2001)</th>
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<tr>
<td><strong>Parameters</strong></td>
<td><strong>Notation</strong></td>
<td><strong>Value</strong></td>
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<td>job-finding rate elasticity with respect to market tightness</td>
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<td>exponential distribution parameter</td>
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**Key variables**

| informal vacancies | $v_i$ | endogenous | equation (15) |
| unemployment | $u$ | endogenous | equation (16) |
| entrepreneurial ability threshold | $R$ | endogenous | equation (8) |
| labour market tightness | $\theta$ | endogenous | equation (9) |

**Driving forces in business cycle**

| productivity of the match | $\rho$ | stochastic | simulation |
| job destruction rate | $\delta$ | stochastic | simulation |

*As in Boeri and Garibaldi (2006), we assume that $\beta$ and $\eta$ are identical in the two sectors. Furthermore, we assume that the entry cost in the informal sector is equal to zero; indeed, according to condition (A.1') and using the parameters of table 1, we will get a negative entry cost in the informal sector.*
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*Table 2. Results*