Università di Torino

Scuola di Dottorato in Scienze Umane e Sociali

Dottorato di Ricerca in Economia “Vilfredo Pareto”

Measure Theory

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Instructor
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Course Description

The course introduces to the theory of Lebesgue integration. After defining appropriate classes of subsets of a given non-empty set as the basic framework, we provide the notion of Lebesgue-Stieltjes measures and results on their extension to general classes of subsets. We then define constructively integrals with respect to a Lebesgue-Stieltjes measure and state their most relevant properties. The relationship with classical Riemann integration and the connection with probability theory are also examined. Finally, if time allows the introduced notions will be extended to more than one dimension.

Exam

Final grades are determined by a written exam on both theory and exercises.

Course Outline

- Classes of subsets: algebras, semi-algebras, sigma-algebras, monotone classes.
- Measures: definition and properties, finite- and sigma-additivity; construction of measures on sigma algebras; completions of measures; Lebesgue-Stieltjes measures.
- Measurable functions: definition and examples; construction of measurable functions as limits.
- Lebesgue-Stieltjes integrals: construction and properties.
- Convergence theorems: monotone convergence, dominated convergence.
- Null-measure sets and properties holding almost everywhere.
- Comparison with Riemann integration
- Radon-Nikodym’s Theorem
- Integration on product spaces and Fubini-Tonelli’s Theorem

Textbooks

There is no required textbook for the course, the material taught in class being sufficient for the exam. Some reference books are:

- Billingsley, P. (1986). *Probability and measure*. Wiley.
- Dudley, R.M. (2004). *Real analysis and probability*. Cambridge University Press.
- Folland, G.B. (1999). *Real analysis*. Wiley.
- Kolmogorov, A.N. and Fomin, S.V. (1975). *Introductory real analysis,* Dover.
- Royden, H.L. (1988). *Real analysis.* Prentice Hall.
- Rudin, W. (1976). *Principles of mathematical analysis*. McGraw-Hill.
- Rudin, W. (1970). *Real and complex analyis*. McGraw-Hill.